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EXPERIMENTAL AND ANALYTICAL STUDY ON THE DYNAMIC BEHAVIOR OF FRAMES UNDER RELATIVE DISPLACEMENT BETWEEN SUPPORT POINTS

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ABSTRACT

This study is concerned with the feasibility of the equation of motion considering dynamic relative displacement between support points. It provides an experimental result of a testing frame structure under dynamic relative displacement and simulation of the dynamic behavior using the equation. The results showed that the simulation could represent the dynamic behavior of the frame under relative displacement between support points and it could be concluded that the analytical method could represent the effect of both internal force and relative displacement. Based on the investigation, non-linear analyses of a rigid-frame bridge were carried out. According to the results, it was found that the amount of relative displacement between support points affected the extent of bridge damage.

KEYWORDS: Equation of Motion, Inertial Force, Relative Displacement, Non-Linear Dynamic Response Analysis

INTRODUCTION

The author has investigated the influence of the relative displacement between support points of a bridge to the seismic response using an analytical method. The method is based on an equation of motion which can represent the effect of both inertial force and relative displacement. The method shows the possibility of solving the dynamic behavior of a bridge concerned with relative displacement between its support points. This method was originally developed for the evaluation of the inelastic behavior of a bridge under time-dependent relative displacement with acceleration, such as a fault movement between the support points or dynamic relative displacement caused by soil liquefaction. However, it is obvious that two or more locations are shaken by different vibration when an earthquake hits the area. Therefore we should consider the effect of relative displacement such as one with a phase difference or with different seismic waves, when we try to represent dynamic behaviors of bridges with dynamic relative displacement. In other words, even in the elastic response, it is very important for the bridge design to represent a bridge behavior under dynamic relative displacement.

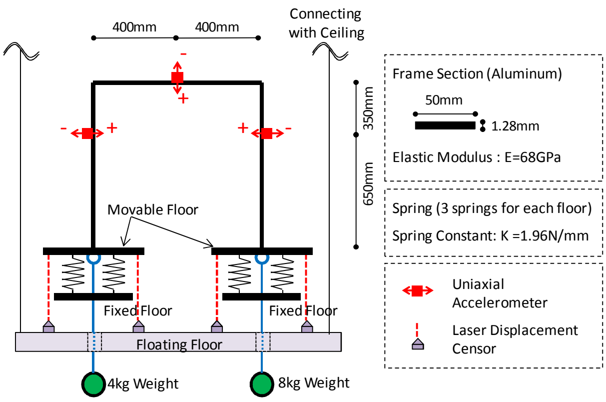


Figure 1: Testing Frame

This study provides an experimental result of frame structure subjected to dynamic relative displacement, and simulation of its dynamic behavior using the equation of motion developed by the author. Based on the investigation, analyses of a rahmen bridge were carried out and the effect of the amount of the relative displacement was discussed.

OUTLINE OF TEST

Testing Frame

To observe the influence of both inertial force and relative displacement of support points, a frame model and 2 vibration floors were prepared. The frame members were aluminum plates having sections of 5mm width and 1.28mm thickness. The frame has been assembled as shown in Figure 1. The measurements of the frame are also found in the figure. The connections of members are joined and fixed to each other by steel angles.

Each vibration floor was composed of 2 acrylic plates and 3 springs. The springs were put between the plates, and both ends of each spring were fixed. The spring constant of each spring can be found in Figure 1. Note that we also put 3 cylinders between the plates unless the support plates generate excess of rotation.

The frame model and the vibration floors were put on a fixed stand. One of the vibration floors (fixed floor in figure 1) was fixed with the stand. After the installation of them, we connected weights with 2 vibration floors by strings, and we can generate shaking at the base of the frame by pulling down the weights.

Measurements of Displacement and Acceleration

For each vibration floor, two laser displacement censors were placed parallel on the floating floor hung from the ceiling to measure the time history displacements of the vibration floor. The distances between 2 censors were L=345mm (Left) and L=350mm (Right). The floating floor was hung from the ceiling because it is necessary to avoid interference with the vibration which was generated by the impact caused by the vibration floors.

Giving the displacements of d_A[mm] and d_B[mm], the central displacement at the support point d [mm] and its rotation θ [rad] become

$$d = \frac{d_A + d_B}{2} \quad , \theta = \frac{d_B - d_A}{L}$$

We also put 3 accelerometers (22.5g each) on the frame. These accelerometers are for the measurements of the response of the frame. The location of the accelerometers is shown in Figure 1.

ANALYSIS OF TESTING FRAME VIBRATION

Formulation of the Equation of Motion with Relative Displacements

The equation of motion for a whole structure in the absolute coordinates without the effect of damping is expressed as below.

$$M\ddot{u} + Ku = F$$

Here, M is the mass matrix, K the stiffness matrix, u absolute displacement vector and F the external force vector. We divide these matrices and vectors as

$$M = \begin{bmatrix} M_s & 0 & 0 \\ 0 & M_A & 0 \\ 0 & 0 & M_B \end{bmatrix} \quad K = \begin{bmatrix} K_{ss} & K_{SA} & K_{SB} \\ K_{AS} & K_{AA} & K_{AB} \\ K_{BS} & K_{BA} & K_{BB} \end{bmatrix} \quad u = \begin{Bmatrix} u_s \\ u_A \\ u_B \end{Bmatrix} \quad F = \begin{Bmatrix} F_s \\ F_A \\ F_B \end{Bmatrix} \tag{1}$$

Here, the subscripts A,B mean the degrees of freedoms corresponding to the support points A,B and S the degree of freedoms without any supports. Considering the condition when the support points displace u_A and u_B statically, we have

$$\begin{bmatrix} K_{ss} & K_{SA} & K_{SB} \\ K_{AS} & K_{AA} & K_{AB} \\ K_{BS} & K_{BA} & K_{BB} \end{bmatrix} \begin{Bmatrix} u_{s0} \\ u_A \\ u_B \end{Bmatrix} = \begin{Bmatrix} F_{s0} \\ F_A \\ F_B \end{Bmatrix} \tag{2}$$

Here, u_{s0} is the displacements of the load control points, F_{s0} the stationary loads and F_A (F_B) is the reaction force due to the displacement u_A (u_B).

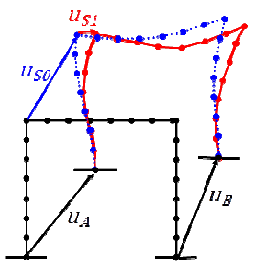


Figure 2: Division of Displacement Vector

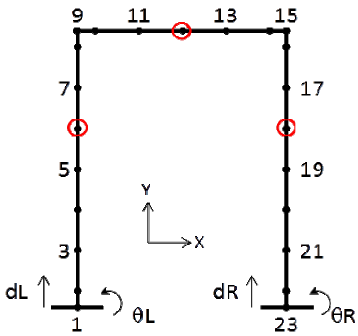


Figure 3: FE Model

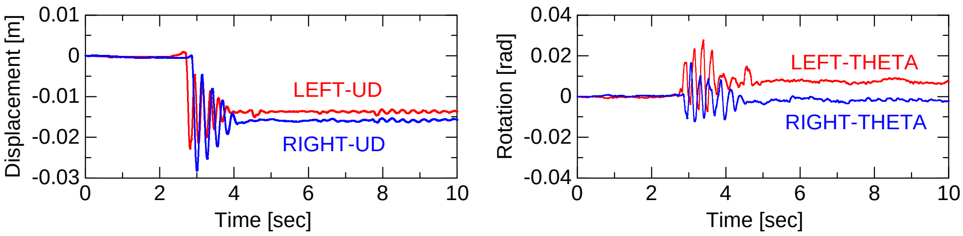


Figure 4: Observed Displacement and Rotation

Expanding the first row, we have

$$K_{ss}u_{s0} + K_{SA}u_A + K_{SB}u_B = F_{s0} \tag{3}$$

Next, we introduce u_{s1} which is the displacement vector due to the dynamic effects at load control points (Figure 2). Using the relations $u_s = u_{s0} + u_{s1}$ and $F_s = F_{s0}$, Eq.(1) yields

$$M_s(\ddot{u}_{s0} + \ddot{u}_{s1}) + K_{ss}(u_{s0} + u_{s1}) + K_{SA}u_A + K_{SB}u_B = F_{s0} \tag{4}$$

Substituting Eq.(3) into Eq.(4), we obtain

$$M_s\ddot{u}_{S1} + K_{SS}u_{S1} = -M_s\ddot{u}_{S0} \tag{5}$$

Then, Eq.(3) can be rewritten as

$$u_{s0} = K_{SS}^{-1}(F_{S0} - K_{SA}u_A - K_{SB}u_B) \tag{6}$$

And therefore Eq.(5) becomes

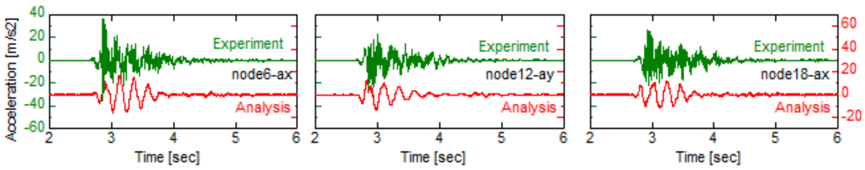
$$M_s\ddot{u}_{S1} + K_{SS}u_{S1} = M_sK_{SS}^{-1}(K_{SA}\ddot{u}_A + K_{SB}\ddot{u}_B) \tag{7}$$

In Eq.(7), the $u_{s1}\,(\ddot{u}_{s1})$ are the relative displacement (acceleration). Consequently, we need to add $u_{s0}\ (\ddot{u}_{s0})$ to obtain the absolute displacement (acceleration), that is,

$$u_S = u_{S0} + u_{S1}\,,\ \ \ddot{u}_s = \ddot{u}_{S0} + \ddot{u}_{S1} \tag{8}$$

Analytical Model of Testing Frame

A beam element model was prepared to simulate the phenomenon of the testing frame. The model was a lumped mass model with 23 nodes and 22 elements. To simulate the dynamic behavior of the frame, the observed time-history displacements and rotations were inputted at both support points (1st and 23rd nodes). The observed displacement and rotation time-history can be found in Figure 4



a) Node6 / X-direction b) Node12 / Y-direction c) Node18 / X-direction

Figure 5: Observed Accelerations and Analytical Results

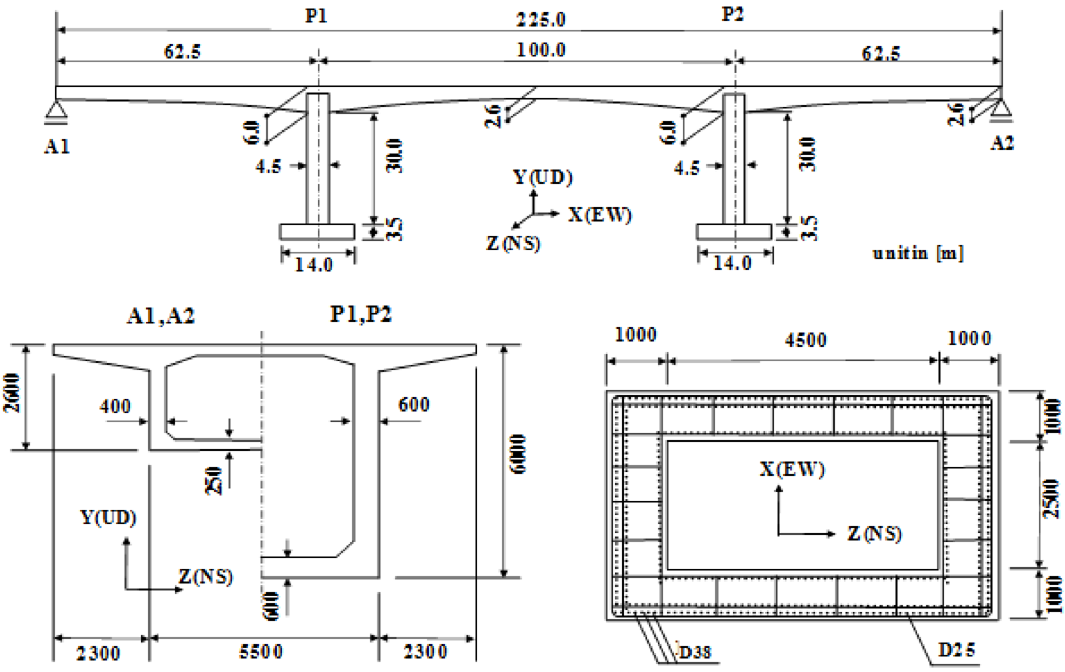


Figure 6: Target PC Rahmen Bridge

Analytical Results

The analytical results compared with the test results are shown in Figure 5. All the acceleration waveforms obtained from the analysis are similar to observed ones. In comparison a) with c), opposite phase from the different time-history displacements (Figure 4) were represented by the analysis. From these results, the formulation in Chapter 2 can represent the dynamic effect of the relative displacement between support points.

APPLICATION OF THE METHOD TO A RAHMEN BRIDGE

Incremental Form of the Equation of Motion and Solution Method

To apply Eq.(5) to non-linear analysis, we need to express the equation in the incremental form. The incremental form of Eq.(5) becomes

$$M_S\Delta\ddot{u}_{S1} + K_{SS}^t\Delta u_{S1} = -M_S\Delta\ddot{u}_{S0} \tag{9}$$

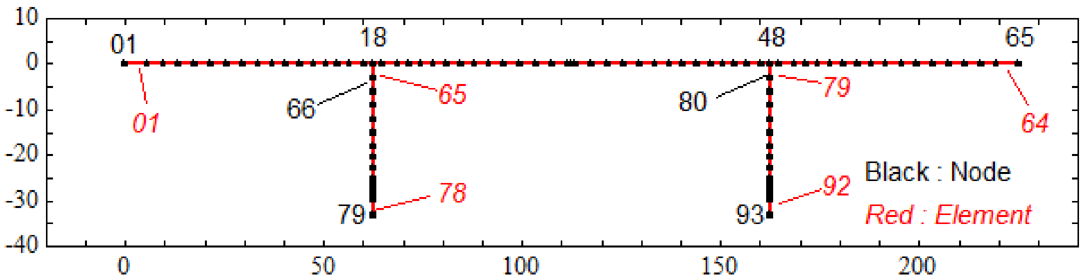


Figure 7: Beam Element Model

Table 1: Material Properties

Concrete	Beam	Design Strength	$f'_{ck} = 40\text{MPa}$
		Initial Elastic Modulus	$E_c = 31\text{GPa}$
	Pier	Design Strength	$f'_{ck} = 27\text{MPa}$
		Initial Elastic Modulus	$E_c = 26.5\text{GPa}$
Reinforcing Steel		Yield Strength	$f_y = 345\text{MPa}$
		Elastic Modulus	$E_s = 200\text{GPa}$
PC		12S12.7 SWPR7B	

$\Delta\ddot{u}_{S0}$ in the right hand side is the increment of the second order differential of u_{S0}. u_{S0} can be obtained by solving Eq.(2). Therefore, we can calculate the right hand side of Eq.(9) using the u_{S0} in the previous step and the u_{S0} before the previous step in the time-history response analysis. Once we obtain$\Delta\ddot{u}_{S0}$, we can analyze the non-linear response of the target structure considering both the effects of the acceleration and the relative displacements by using Eq.(9). The algorithm of time history response analysis can be found in Ref. [1][2].

Target PC Rahmen Bridge

A PC rahmen bridge shown in Figure 6 was analyzed to evaluate damage due to fault movements in this study. The bridge was idealized as 93 degrees of freedom lumped mass system (Figure 7). The moment-curvature relationships were obtained by the fiber model. In the fiber model, material models are applied to concrete, reinforcing bars and PCs. The constitutive relation proposed by Maekawa and Tsuchiya [3] was used for concrete cells, the ones for reinforcing bars and PCs were bilinear model. Material properties are shown in Table 1.

The Ground Displacements from Accelerometer Records

It is necessary to use ground displacement responses in the above mentioned formulation. However, when we integrate time-history acceleration directly, the velocity and displacement time-histories diverge. The reason is the background noise caused by electrical acceleration records. To overcome the divergence, Ohta and Aydan [4] suggested the EPS method which is the displacement calculation method. In this method, we screen the effects of the erratic pattern from electrical acceleration records to integrate acceleration records and hence, we can control divergence phenomena of integration of acceleration.

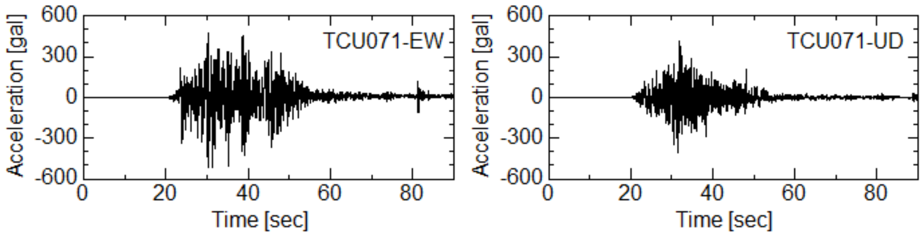


Figure 8: Ground Acceleration of 921Chi-Chi Earthquake at TCU071 in Taiwan (1999)

Table 2: Baseline Offsets [S], EPS Parameters [Gal] and Residual Displacements [M]

	t_1	t_2	ε_1	ε_2	Residual Disp.
TCU071EW	35.4	65.0	2.0	5.0	-1.374
TCU071UD[A]	20.0	55.0	2.0	20.0	+2.148
TCU071UD[B]	20.0	60.0	2.0	20.0	+2.351
TCU071UD[C]	20.0	65.0	2.0	20.0	+2.512

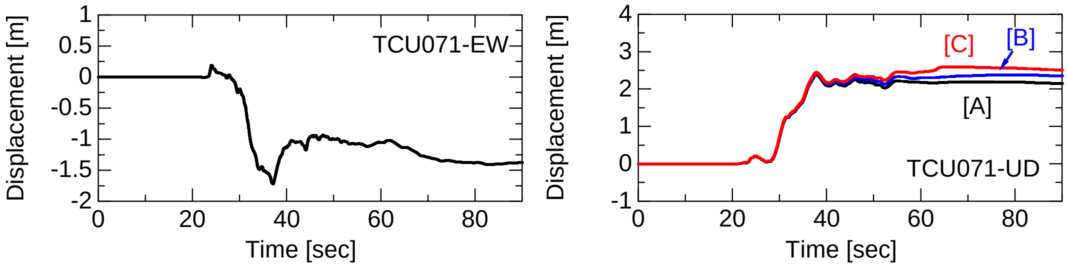


Figure 9: Displacement Time History Obtained by EPS Method

Figure 8 shows the acceleration observed in the Taiwan Chi-Chi earthquake, 1999 (Taiwan Central Weather Bureau, TCU071[5]). Integrating these acceleration records by EPS method using the parameters shown in Table 2, we can obtain several displacement responses as shown in Figure 9.

Analytical Results and Discussions

To examine the effect of dynamic horizontal relative displacement between support points, 3 analyses were carried out.

Case A: input TCU071UD [A] at all the support points (A1,P1,P2 and A2)

Case B: input TCU071UD [A] at A1 and P1 / input TCU071UD[B] at P2 and A2).

Case C: input TCU071UD [A] at A1 and P1 / input TCU071UD[C] at P2 and A2).

In all the cases, TCU071EW is also inputted at all the support points. Therefore, the final relative displacement are 0.000m for case A, +0.203m for case B and +0.364m for case C.

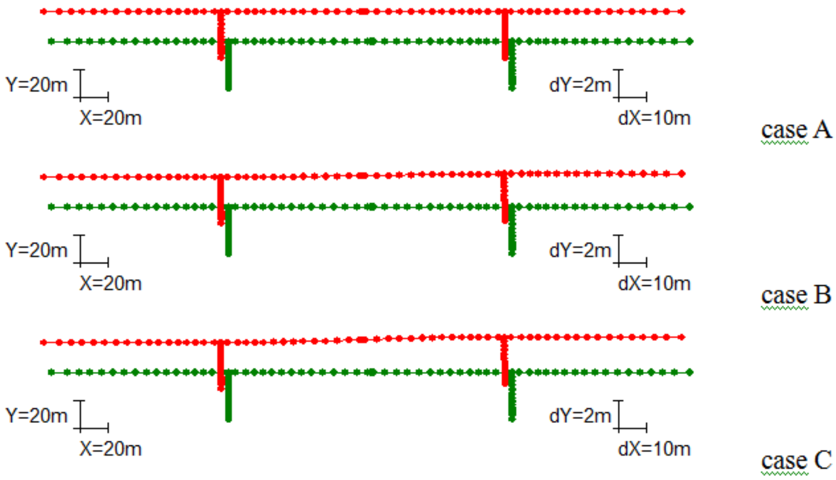


Figure 10: Analytical Results (Final Deformation of the Bridge)

Figure 10 shows the final deformations of all the cases. The result of case A shows the same displacements of the support points and therefore this is the result of the bridge behavior under uniform acceleration. The deformation of case B and C shows that a large deformation appears in the main span of the bridge and also shows the deformation in the side spans are similar to that of case A.

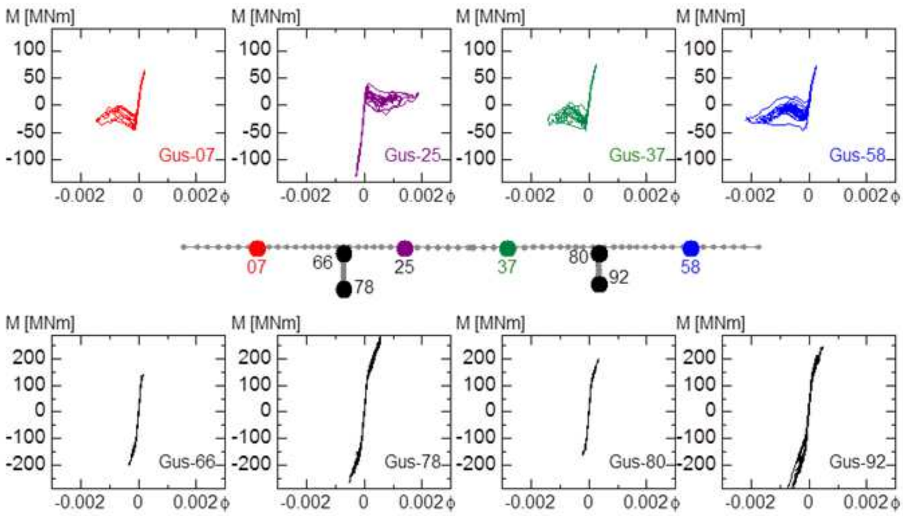


Figure 11: Moment-Curvature Relationships in the Beam and Piers (Case A)

To investigate these results, the moment-curvature(M-□) relationships in the beam and piers are shown in Figure 11 to 13. Figure 11 shows the M-□ relationships of case A. The figure shows that no plastic hinges are generated on the piers. However, plastic hinges are generated on both the main span and the side spans.

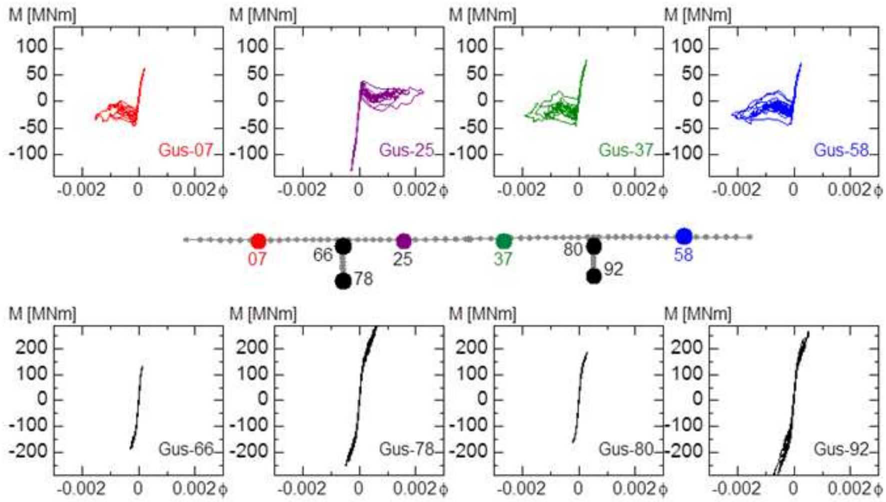


Figure 12: Moment-Curvature Relationships in the Beam and Piers (Case B)

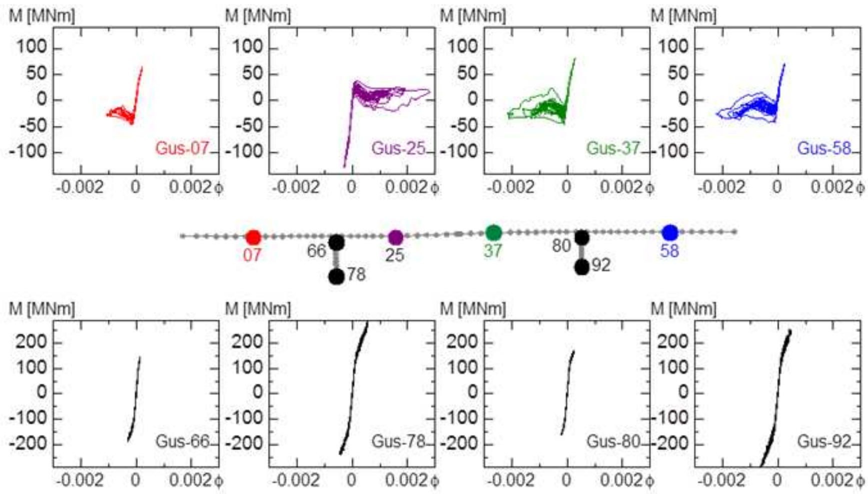


Figure 13: Moment-Curvature Relationships in the Beam and Piers (Case C)

Figure 12 shows the M-φ relationships of case B. The figure also shows the same tendency as that of case A. The maximum response curvatures in main span are greater than that of case A, though the maximum response curvatures in side spans are almost the same as that of case A.

Figure 13 shows the M-φ relationships of case C. The maximum response curvatures in main span are greater than that of case B, However those in side spans are almost the same as that of case A.

The reason of the results is that the relative displacement generates between P1 and P2. The side spans are located at between A1 and P1 (or between P2 and A2), and no relative displacement had been generated at the side spans. Therefore the larger the relative displacement becomes, the more only the main span is damaged.

In addition to this, these results imply that the damage considering dynamic effect and relative displacements (case B/C) will be more severe than the damage of the bridge under only dynamic effect (case A). Hence, it can be concluded that it is quite important to consider the effect of inertial force when we consider the damage of a bridge under fault movements.

CONCLUSIONS

An experiment was carried out to discuss the applicability of the equation of motion considering both dynamic relative displacement and inertial force in this paper. This paper also shows the effect of vertical relative displacement to a rahmen bridge.

The elastic analysis using the method could represent the experimental result and it can be concluded that the method can represent the dynamic effect of the relative displacement between support points.

From the non-linear analysis of a bridge, it was shown that there are some cases in which we should consider the interaction of relative displacement and inertial force to predict the damage of bridges.

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